SUBTRACTION Stage 1(i) : Using the empty number line to count on in jumps of one

Children must be happy with using number lines and be able to visualise how a number line can be started from other points than 0 before they are introduced to this method. Similarly, as they work beyond 20 they need to be able to count to 100 confidently and be able to count on and back in Tens. Seeing the link and talking about what they have noticed when adding or subtracting Tens to TU numbers.

Subtraction by counting on rather than counting back or taking away by using an open number line.
In the early stages (Y1) children will be taught how to solve subtraction problems by using an empty number line to count on making jumps of one.
Each time children need to jump from the number they are on to the next marking the jump and recording the number where they land and the size of the jump. Time invested in practice and explicit teaching at this stage will ensure that children really understand how the empty number line is used.
Model explicitly how the method works – subtracting by counting on.
Children need lots of practical play experiences with money, etc to help them see the link between counting on and subtraction.

10 – 5 =

Teacher modelling needs to include:

- asking children where to start number line and how far we are counting on to complete our subtraction
- drawing open number lines starting at the number subtracted and ending with the number being subtracted from
- focus on the number they are counting on to as they get used to making jumps along an open number line
- clear and repeated demonstration of counting the jumps made to complete the subtraction calculation

SUBTRACTION Stage 1(ii) : The empty number line using bigger jumps

This stage explicitly models and practises how Number Bonds can be applied to help make calculations more efficient. Also how knowledge of counting on in Tens (the Units digit stays the same but the Tens digit gets one Ten bigger) can be effectively applied to calculations.

The next step is to model how the number bonds to 10 that are learnt and regularly practised can be applied to make quicker jumps on the number line, e.g. Lots of work showing how a jump of 6 starting from 4 will always land on 10 – this can be illustrated and reinforced with activities and games on a hundred square. This is the way to help children see that they can always jump from the Units to the next multiple of Ten (Tens number) by applying number bonds to 10 – this needs explicitly modelling and teaching so that all children make the link and apply their knowledge.

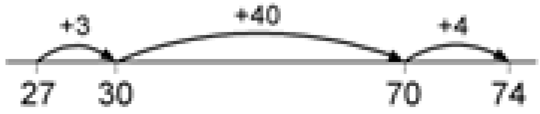
Similarly, children who have learnt to count on in Tens and to see in terms of place value what is happening will soon make jumps on to the next multiple of 10.

15 - 8 =

the number line starts with the number to be subtracted, counting on to the larger number from which it is subtracted, to find the difference between the two numbers

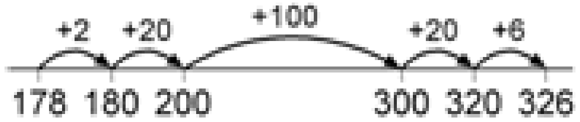
The counting-up method

- The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ? = 74 mentally.



The counting-up method with three digit numbers

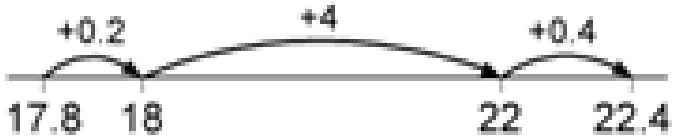
- With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ? = 200 and 200 + ? = 326 mentally.
- The most compact form of recording remains reasonably efficient



or

The counting-up method with decimals

- The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.
- This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.



SUBTRACTION Stage 2(i) : Partitioning TU without exchanging

The next stage is to record quick mental methods using partitioning and can be begun quickly once children are sure of place value in terms of Tens and Units. In the early stages teachers need to ensure there are no cases where exchange of Tens for Units is required to complete the subtraction. This stage parallels the Partitioning for addition but uses a vertical rather than a horizontal layout.

It is apparent how important it is to keep children regularly rehearsing and practising number bonds to make sure that they can apply speed of recall to 4 - 3 and so to 4T-3T (40-30), etc. It may be important to practise subtraction facts MORE than addition facts and explicitly make the links between addition and subtraction. Although if some children prefer the open number line and are happier using an open number line using stronger addition facts and skills this method should be available for them to solve problems and answer calculations.

Partitioning numbers into Tens and Units, and writing them vertically mirrors the column method, where Units are placed under Units and Tens under Tens.

44 - 31 =

	Tens	Units
	40	4
-	30	1
	10	3

so 44 - 31 = 13

Generally children find counting on/adding faster and easier than counting back/subtracting and it is positive to model counting on from the smaller number – mirroring the method for open number lines.

So, instead of saying "what's 40 subtract 30?" you can model counting up /adding from the number being subtracted "what do I count on/add to get from 30 to 40", working from the subtracted number and counting on/adding

SUBTRACTION Stage 2(ii) : Exchanging a Ten for 10 Units

In preparation for full decomposition where it is necessary to change a Ten for Ten Units children in Year 4 should start to explore, practise and rehearse exchanging 1T for 10U, recording it by striking out the original Tens digit and recording it as 1 less, whilst changing the Units digit by adding the 10U exchanged.

This can be well modelled and explained in the context of money and to secure understanding children can be given plenty of opportunities to exchange 10p for ten 1p coins to see the effect exchanging 10p for ten 1p coins has

Exchanging a Ten for 10 Units

$$\begin{array}{cc} \mathbf{T} & \mathbf{U} \\ \mathbf{4} & \mathbf{3} \end{array} \longrightarrow \begin{array}{cc} \mathbf{T} & \mathbf{U} \\ {}^{3}\cancel{\mathbf{4}} & {}^{1}\mathbf{3} \end{array}$$

43p - four 10p coins and three 1p coins
one 10p coin can be exchanged for ten 1p coins
giving three 10p coins and thirteen 1p coins

SUBTRACTION Stage 3: Expanded layout using Partitioning and Exchanging

The expanded method leads children to the more compact method so that they understand its structure and efficiency.

The concept of exchanging is vital for decomposition.The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Teachers should begin with TU numbers then move on to HTU, THTU, decimals, etc, modelling and rehearsing exchanging at every stage.

Teachers need to be aware that it may be quicker and more efficient to model to children how to count on from the number being subtracted rather than subtracting to find the answer. Avoid making statements that are not mathematically correct e.g. the old way of saying “1 subtract 7 I can’t do, so I have to exchange....” is mathematically not correct because 1 subtract 7 is -6

Rather start at the number to be subtracted and count up, therefore from 7 I can’t count up to 1, so I need to exchange. Once the exchange is complete ask “how many do I add to get to 11 from 7?” rather than “11-7” and make use of known number bonds and facts and the relationship between addition and subtraction.

Examples should be used to demonstrate to children how they can choose the most appropriate and efficient strategy for a subtraction calculation. In this case it may be quicker and more efficient to use a number line to avoid lots of exchanging.

Partitioned numbers are written under one another:

Example: 74 - 27

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}} \\ -\ 20+7 \\ \hline 40+7 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

Example: 741 - 367

$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

SUBTRACTION Stage 4 : Expanded method leading to column subtraction

Example: 563 − 241, no adjustment or decomposition needed

Expanded method

$$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array} \qquad \text{leading to} \qquad \begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$$

Start by subtracting the Units then the Tens, then the Hundreds. Refer to subtracting the Tens, for example, by saying what gets me from forty to sixty, not 'six take away four'.

Example: 563 − 271, adjustment from the Hundreds to the Tens, or partitioning the Hundreds

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+1 \\ \hline \end{array} \qquad \begin{array}{r} 400+160+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array} \qquad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{160}{\cancel{60}}+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{16}{\cancel{6}}\ 3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the Hundreds, Tens and Units components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the Tens becomes '160 minus 70', or count on from 7T to 16T – what goes with 7 to make 16 – using number bonds, an application of subtraction of multiples of ten by recall of number bonds within 20.

Example: 563 − 278, adjustment from the Hundreds to the Tens and the Tens to the Units

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+150+13 \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array} \qquad \begin{array}{r} \overset{\overset{400}{\cancel{500}}}{\cancel{500}}+\overset{\overset{150}{\cancel{50}}}{\cancel{60}}+\overset{\overset{13}{13}}{\cancel{3}} \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{15}{\cancel{6}}\ \overset{13}{\cancel{3}} \\ -\ 2\ 7\ 8 \\ \hline 2\ 8\ 5 \end{array}$$

Here both the Tens and the Units digits to be subtracted are bigger than both the Tens and the Units digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

$$\begin{array}{r} 500+0+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \qquad \begin{array}{r} \overset{\overset{400}{\cancel{400}}}{\cancel{500}}+\overset{\overset{90}{\cancel{100}}}{\cancel{0}}+\overset{\overset{13}{3}}{\cancel{3}} \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{9}{\cancel{0}}\ \overset{13}{\cancel{3}} \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.